

As Level Physics - Flying Start

Mathematical concepts

You should be able to:

- Perform calculations in standard form
- Convert between standard and decimal form
- Use a calculator to express numbers in standard form
- Use common physical constants in standard form

The standard form expresses a number as $a \times 10^b$, with $0 < a < 10$ and b a +/- integer.

For example

$$F = 1.63 \times 10^3 \text{ N}$$

In physics many important constants are either very large or very small, the standard form is a convenient way of expressing these numbers. Using standard form also reduces calculation errors.

You are expected to express final results in standard form and to be able to convert to and from decimal form. The force above can be expressed as

$$F = 16.3 \times 10^2 \text{ N is not acceptable}$$

$$F = 0.163 \times 10^4 \text{ N is not acceptable}$$

$$F = 1630 \text{ N is not acceptable}$$

$$F = 1.63 \text{ kN is acceptable}$$

While the first three expressions above are numerically equivalent to the standard form above, these are not acceptable standard form. The final expression, using the prefix *kilo* is acceptable.

Calculator Use

With access to a scientific calculator you should be able to use it to convert between different decimal/standard form calculations. For Sharp calculators the 'Change' button is used whilst for Casio it is the "S→D" button that performs this operation. You should be encouraged to use this calculator function to help with the accuracy of their answers.

To write numbers in standard form on a calculator either use a "×10^x" button (Casio) or the "EXP" button (Sharp).

For other models please encourage You to investigate for themselves the appropriate functions.

Contexts in physics

- Any calculations which involve large or small numbers will require the use of standard form.

A lot of calculations in Physics involve large and small numbers. Formulae involving Newton's gravitational constant, Boltzmann's constant, Planck's constant, the speed of light, the fundamental electron charge and the masses of large /small bodies are typical examples of this. Common physical constants used are found in the data, formulae and relationships booklet and should be used to the accuracy stated unless otherwise specified.

For example in calculating the gravitational force between the Earth and the Sun, Newton's gravitational constant is required; $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. The mass of the Earth is $5.79219 \times 10^{24} \text{ kg}$, the mass of the Sun is $1.989 \times 10^{30} \text{ kg}$ and the *mean* distance between them is $1.496 \times 10^{11} \text{ m}$.

Applying the Newton's law of gravitation $F = GMm/r^2$

$$\text{gives: } F = 6.67 \times 10^{-11} \times 5.79219 \times 10^{24} \times 1.989 \times 10^{30} / (1.496 \times 10^{11})^2$$

This can be worked out one step at a time using the power laws.

Notice that the 1.496 also has to be squared

$$F = 76.8428416197 \times 10^{43} / 2.238016 \times 10^{22}$$

As Level Physics - Flying Start

Finally using the division law: $F=34.3352512313 \times 10^{21} \text{N}$

This can only be stated to 3 significant figures and needs to be put in standard form.

Therefore $F=34.3 \times 10^{21}=3.43 \times 10^{22} \text{N}$

M1.1 Use an appropriate amount of significant figures

Mathematical concepts

You should be able to:

- Be able to round to a given number of significant figures or decimal places
- Give a result of a calculation to a level of accuracy appropriate to the level of the inputs

You must understand that the lowest level of significant figures in the inputs of a calculation will determine the number of significant figures in the answer. If there are 3 inputs to a particular calculation and they are quoted as being correct to respectively 2, 3 and 4 significant figures then the answer can only be quoted to 2 significant figures.

You will be familiar on rounding numbers in other contexts and in itself this should pose few problems. It is important to recognise that a zero at the end of a number is considered a significant figure.

For example, rounding 4.99×10^5 to 2 significant figures would yield 5.0×10^5 . The 0 in this context represents a significant figure, not a place holder, and should be retained. Omitting the zero and quoting the answer as 5×10^5 is incorrect, as the result now only has one significant figure.

Contexts in Physics

You will be expected to:

- Report calculations to an appropriate number of significant figures given raw data quoted to a varying number of significant figures.
- Understand that calculated results can only be reported to the limits of the least accurate measurement.

Example

The electron mass is given to 1sf as 9×10^{-31} and the speed of light is given to 3sf as 3.00×10^8 . In calculating the rest energy using $E=mc^2$, the answer can only be given correctly to 1 significant figure as this is the least accurate measurement;

$$E=9 \times 10^{-31} \times 9.00 \times 10^{16} = 81 \times 10^{-15} = 8.1 \times 10^{-14} = 8 \times 10^{-14} \text{ J.}$$

You also need to be aware that when making measurements that these issues show themselves through the accuracy of your measuring tools.

Uncertainties

In performing practical work, You should be aware that uncertainties are inherent in any measurement. Any measuring apparatus used has an associated uncertainty.

As a general rule, the uncertainty is often taken to be half a division on either side of the smallest unit on the scale you are using. However, the accuracy of measurements does also depend on the quality of the apparatus used, such as a balance, thermometer, caliper or glassware.

For example

As Level Physics - Flying Start

- A 30cm rule has divisions of 1 mm and an uncertainty of half a division or 0.5 mm. When measuring a distance the uncertainty has to be taken into account twice and it is overall 1mm
- A caliper has an accuracy of 0.01 mm when used by a skilled operator

You should be able to calculate from the percentage uncertainty in a given measurement. You should also be aware of the difference in uncertainty of different pieces of equipment. For example, if measuring a length the appropriate measurement device should be chosen, depending on the level uncertainty required.

Examples of uncertainties

Some examples are shown below. Note that the actual uncertainty on a particular item of equipment may differ from the values given below. An item of equipment may have different uncertainties for different range settings.

Ruler

- A ruler with marks every 1mm has an uncertainty of 1mm for a distance measurement
- A caliper has an uncertainty of 0.01 mm when used by a skilled operator

Voltmeter

- A voltmeter has an uncertainty of 0.01V in the 0-20V range
- The same voltmeter might have an uncertainty of 1V in the 0-100V range

Time Measurement

- A stopwatch measures time with a resolution of 0.01s, however the operator reaction time is significantly longer, increasing the total uncertainty in the measurement to perhaps 0.3s
- A light gate measures time with the same resolution of 0.01s, but has a significantly lower total uncertainty as it eliminates the reaction time

Worked examples

The significance of the uncertainty in a measurement depends upon how large a quantity is being measured. It is useful to quantify this uncertainty as a percentage uncertainty

percentage uncertainty = $\frac{\text{uncertainty} \times 100\%}{\text{quantity measured}}$

For example, a two-decimal place balance may have an uncertainty of 0.005 g.

For a mass measurement of 2.56 g

- percentage uncertainty = $(0.005 / 2.56) \times 100\% = 0.20\%$

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For a mass measurement of 0.12 g, the percentage uncertainty is much greater

- percentage uncertainty = $(0.005/0.12) \times 100\% = 4.2\%$

Multiple measurements

Where quantities are measured by difference, there will be an uncertainty in each measurement, which must be combined to give the uncertainty in the final value. The principle of the following example can be applied to other quantities measured by difference.

The difference in length of a rod due to a change in temperature is to be found. The absolute uncertainties of both measurements are summed up to give the uncertainty in the change in length.

Using a rule to determine the elongation of a metal rod due to thermal expansion

Length when cold = 54.3 cm uncertainty = 0.1 cm

Length when hot = 55.2 cm uncertainty = 0.1 cm

Increase in length = 0.9 cm overall uncertainty = 2×0.1 cm

percentage uncertainty in the elongation = $((2 \times 0.1)/0.9) \times 100\% = 22\%$

While there is a negligible percentage uncertainty in each length measurement, the overall percentage uncertainty in the elongation is much greater and care should be taken to ensure the measurement technique and apparatus are appropriate.

Note

Some textbooks available do not give a consistent message regarding the treatment of uncertainties. In Physics A and B we will therefore allow both half the smallest division as the absolute uncertainty for a measuring instrument and the smallest division itself as the absolute uncertainty. This will ensure that we do not penalise candidates in any examination - since this ambiguity is not your fault.

Recording measurements

When using a digital measuring device (such as a modern top pan balance or ammeter),

- record *all* the digits shown (Note: when using a digital timer such as a stopwatch, do not record to more than two decimal places.)

When using a non-digital device (such as a ruler or a measurement cylinder),

- record all the figures that are known and, where appropriate, an additional estimated figure may be allowed

Reading a ruler

A ruler is graduated in divisions every 1 mm. A ruler is a non-digital device, so we record all figures that are known for certain. We can estimate a further figure.

Using the half-division rule, the estimation is 0.5 mm. The overall uncertainty in any distance measured always comes from two measurements, so the overall uncertainty = 2×0.5 mm = 1 mm.

In a distance measurement covering the entire 300 mm length of the ruler, the uncertainty is small
percentage uncertainty = $(0.5 \times 2 / 300.0) \times 100\% = 0.3\%$

As Level Physics - Flying Start

For shorter distances, the percentage uncertainty becomes more significant. For measuring a distance of 25 mm:

$$\text{percentage uncertainty} = (2 \times 0.5 / 25.0) \times 100\% = 4$$

Mean values

When calculating the mean value of measurements, it is *acceptable* to increase the number of significant figures by 1.

Percentage Difference

You may be asked to determine the difference between experimental values and accepted values. 'Experimental values' are those that are derived from measurement or calculation, whereas 'accepted' or 'theoretical' values are values that are accepted by the scientific community. The percentage difference between an experimental and accepted value is determined as follows:

$$\text{Percentage difference} = \frac{\text{experimental value} - \text{accepted value}}{\text{Accepted value}} \times 100\%$$

This is also the percentage error.

In many cases there will be no 'accepted value', especially since most experiments are performed to find out something 'new'. However it is considered good practice when developing a new experiment to first try to perform a measurement that does have an accepted value the result can be compared to. The scientist can then assess if your experiment is accurate.

Significant figures

How many significant figures should be used?

The result of a calculation that involves measured quantities cannot be more certain than the *least* certain of the information that is used. So the result should contain the same number of significant figures as the measurement that has the *smallest* number of significant figures.

A common mistake by students is to simply copy down the final answer from the display of a calculator. This often has far more significant figures than the measurements justify.

Rounding off

When rounding off a number that has more significant figures than are justified (as in the example above), if the last figure is between 5 and 9 inclusive round up; if it is between 0 and 4 inclusive round down.

For example, the number 3.5099 rounded to:

4 sig figs is 3.510

3 sig figs is 3.51

2 sig figs is 3.5

1 sig fig is 4

Notice that when rounding you only look at the one figure beyond the number of figures to which you are rounding, *i.e.* to round to three sig fig you only look at the fourth figure.

How do we know the number of significant figures?

When rounding 228.5 to 2 significant figures, an incorrect approach would be to round to 230.

As Level Physics - Flying Start

When seen in isolation, it would be impossible to know whether the final zero in 230 is significant (and the value to 3 sig figs) or insignificant (and the value to 2 sig figs).

In such cases, standard form should be used and is unambiguous:

- 2.3×10^2 is to 2 sig figs
- 2.30×10^2 is to 3 sig figs

When to round off

It is important to be careful when rounding off in a calculation with two or more steps.

- Rounding off should be left until the very end of the calculation.
- Rounding off after each step, and using this rounded figure as the starting figure for the next step, is likely to make a difference to the final answer. This introduces a **rounding error**.

You will often introduce rounding errors in multi-step calculations.

Example

The resistance of a resistor is determined by measuring the potential difference and current. The voltmeter reads 12.0 V and the ammeter 1.3 mA.

The resistance can be found using $R=V/I$.

Using a calculator the resistance is then $12.0/(1.3 \times 10^{-3})=9.2307 \text{ k}\Omega$.

Since the least certain measurement (the current) is only to 2 significant figures, the answer should also be quoted to 2 significant figures.

Therefore, the resistance to the correct number of significant figures is $R=9.2 \text{ k}\Omega$

*It should be noted however, that if this figure is to be used in subsequent calculations then the rounding off should **not** be applied until the final answer has been obtained.*

For example, the resistor is used in a circuit to determine the capacitance of a capacitor. The circuit was found to have a time constant $\tau=RC=0.31 \text{ s}$

Using the calculator value of $9.2307 \text{ k}\Omega$

- $C = 3.3584 \times 10^{-5} \text{ F}$
- rounding to 2 sig figs gives $C = 3.36 \times 10^{-5} \text{ F}$

Using the rounded value of $9.2 \text{ k}\Omega$ to determine the capacitance

- $C = 3.3696 \times 10^{-5} \text{ F}$
- rounding to 2 sig figs gives $C = 3.37 \times 10^{-5} \text{ F}$ and we have a 'rounding error'.

Questions

1 To how many significant figures and decimal places are these numbers stated

- 66 001
- 46.87
- 34 000

As Level Physics - Flying Start

- d) 43 000.00
- e) 0.000 000 47
- f) 3.4×10^9
- g) 7×10^7

2 A box has sides of 34.4 cm what is the surface area and volume (use correct sig figs)

3 Round the following to 3 sig fig

40.03, 40.00, 40.07, 4312, 4318, 432.3, 432.7, 43 330 000

4 A balance measures 467.76 g and the uncertainty is 0.01 g what is the percentage uncertainty?

5 what is the percentage uncertainty if you measure 78.8 cm with a standard metre ruler?

6 A student measure the acceleration due to gravity as 9.76 ms^{-2} what is the percentage error in the measurement if the accepted value is 9.81 ms^{-2} ?

7 A student measures the radius of a ball bearing with an uncertainty of 0.01 mm and finds it to be exactly 2 cm in diameter.

- a) How should the measurement be recorded?
- b) What is the percentage uncertainty?
- c) If the volume of the sphere is calculated, what would happen to the percentage uncertainty? (think carefully about this one!)

As Level Physics - Flying Start

ANSWERS

1 To how many significant figures and decimal places are these numbers stated

- h) 66 001: 5 and none
- i) 46.87: 4 and 2
- j) 34 000: 2 or 5 (ambiguous which is why we use standard form) and none
- k) 43 000.00: 7 and 2
- l) 0.000 000 47: 2 and 8
- m) 3.4×10^9 : 2 and 1
- n) 7×10^7 : 1 and none

2 A box has sides of 34.4 cm what is the surface area and volume (use correct sig figs)

$7.10 \times 10^3 \text{ cm}^2$ or 0.710 m^2 area and $4.07 \times 10^4 \text{ cm}^3$ or 0.0407 m^3 volume

3 Round the following to 3 sig fig

40.03, 40.00, 40.07, 4312, 4318, 432.3, 432.7, 43 330 000
40.0, 40.0, 40.1, 4310 or 4.31×10^3 , 4320 or 4.32×10^3 , 432, 433, 43 300 000 or 4.33×10^7

4 A balance measures 467.76 g and the uncertainty is 0.01 g what is the percentage uncertainty?

0.002%

5 what is the percentage uncertainty if you measure 78.8 cm with a standard metre ruler?

0.127%

6 A student measure the acceleration due to gravity as 9.76 ms^{-2} what is the percentage error in the measurement if the accepted value is 9.81 ms^{-2} ?

-0.510%

7 A student measures the radius of a ball bearing with an uncertainty of 0.01 mm and finds it to be exactly 2 cm in diameter.

- a) How should the measurement be recorded?
- b) What is the percentage uncertainty?
- c) If the volume of the sphere is calculated, what would happen to the percentage uncertainty? (think carefully about this one!)

- a) 2.000 cm or $2.000 \text{ cm} \pm 0.001 \text{ cm}$
- b) 0.05%
- c) 0.15%, volume is radius x radius x radius so the percentage uncertainty is three times larger