

Factorising expressions

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac .
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is $3x^2y$.
So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\text{So } x^2 + 3x - 10 = x^2 + 5x - 2x - 10$$

$$= x(x + 5) - 2(x + 5)$$

$$= (x + 5)(x - 2)$$

- 1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
- 2 Rewrite the b term ($3x$) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4 $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

| | |
|--|--|
| <p>$b = -11, ac = -60$</p> <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$ | <ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms |
|--|--|

Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

| | |
|---|---|
| $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator:</p> <p>$b = -4, ac = -21$</p> <p>So</p> $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$ <p>For the denominator:</p> <p>$b = 9, ac = 18$</p> <p>So</p> $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ <p>So</p> $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$ | <ol style="list-style-type: none"> 1 Factorise the numerator and the denominator 2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) 3 Rewrite the b term ($-4x$) using these two factors 4 Factorise the first two terms and the last two terms 5 $(x - 7)$ is a factor of both terms 6 Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) 7 Rewrite the b term ($9x$) using these two factors 8 Factorise the first two terms and the last two terms 9 $(x + 3)$ is a factor of both terms 10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |
|---|---|

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b $21a^3b^5 + 35a^5b^2$

2 Factorise

a $x^2 + 7x + 12$

c $x^2 - 11x + 30$

e $x^2 - 7x - 18$

g $x^2 - 3x - 40$

b $x^2 + 5x - 14$

d $x^2 - 5x - 24$

f $x^2 + x - 20$

h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$

c $18a^2 - 200b^2c^2$

b $4x^2 - 81y^2$

4 Factorise

a $2x^2 + x - 3$

c $2x^2 + 7x + 3$

e $10x^2 + 21x + 9$

b $6x^2 + 17x + 5$

d $9x^2 - 15x + 4$

f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\frac{2x^2 + 4x}{x^2 - x}$

c $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e $\frac{x^2 - x - 12}{x^2 - 4x}$

b $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d $\frac{x^2 - 5x}{x^2 - 25}$

f $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Hint

Take the highest common factor outside the bracket.

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

- 1 a $2x^3y^3(3x - 5y)$ b $7a^3b^2(3b^3 + 5a^2)$
 c $5x^2y^2(5 - 2x + 3y)$
- 2 a $(x + 3)(x + 4)$ b $(x + 7)(x - 2)$
 c $(x - 5)(x - 6)$ d $(x - 8)(x + 3)$
 e $(x - 9)(x + 2)$ f $(x + 5)(x - 4)$
 g $(x - 8)(x + 5)$ h $(x + 7)(x - 4)$
- 3 a $(6x - 7y)(6x + 7y)$ b $(2x - 9y)(2x + 9y)$
 c $2(3a - 10bc)(3a + 10bc)$
- 4 a $(x - 1)(2x + 3)$ b $(3x + 1)(2x + 5)$
 c $(2x + 1)(x + 3)$ d $(3x - 1)(3x - 4)$
 e $(5x + 3)(2x + 3)$ f $2(3x - 2)(2x - 5)$
- 5 a $\frac{2(x+2)}{x-1}$ b $\frac{x}{x-1}$
 c $\frac{x+2}{x}$ d $\frac{x}{x+5}$
 e $\frac{x+3}{x}$ f $\frac{x}{x-5}$
- 6 a $\frac{3x+4}{x+7}$ b $\frac{2x+3}{3x-2}$
 c $\frac{2-5x}{2x-3}$ d $\frac{3x+1}{x+4}$
- 7 $(x + 5)$
- 8 $\frac{4(x+2)}{x-2}$